

Quantum Gates

Single Qubit Gates

X-gate

$$\boxed{X} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Y-gate

$$\boxed{Y} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Z-gate

$$\boxed{Z} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

H-gate

(AKA Hadamard Gate)

$$\boxed{H} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

T-gate

($\sqrt[4]{Z}$ -gate)

$$\boxed{T} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$

R_ϕ -gate

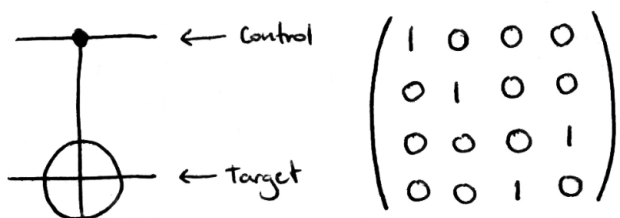
(phase shift)

$$\boxed{R_\phi} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

Two Qubit Gates

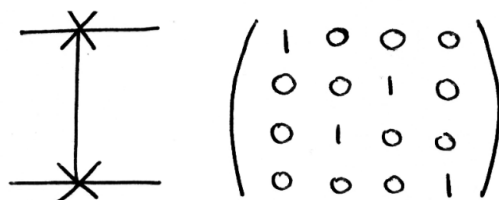
CNOT-gate

Flips the target bit if the control bit is 1

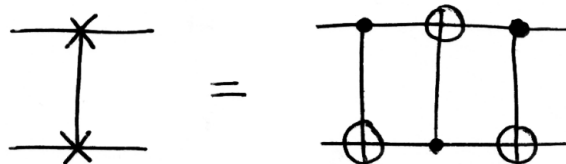
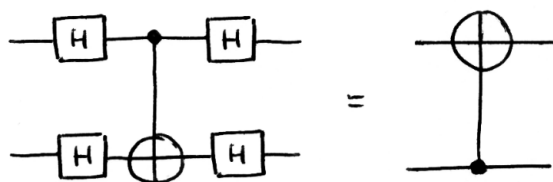


SWAP-gate

Swaps the states of the two qubits
(useful in actual machines)



Circuit Identities



Controls

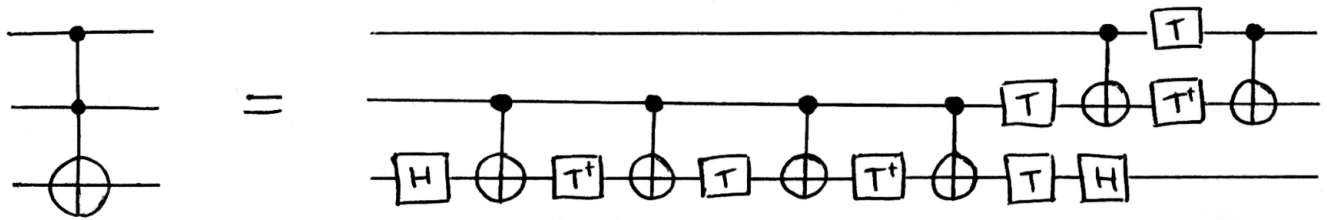
Symbol:

Condition: $|0\rangle$ $|1\rangle$ $|+\rangle$ $|-\rangle$ $|0\rangle$ $|0\rangle$

Quantum Gates

Larger Gates

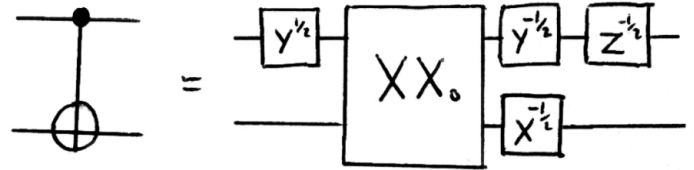
Toffoli Gate



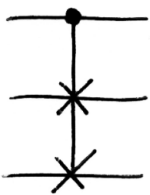
(Toffoli composed from H, T, T[†] & CNOT gates - [Shende & Markov 2008])

Ising Gate

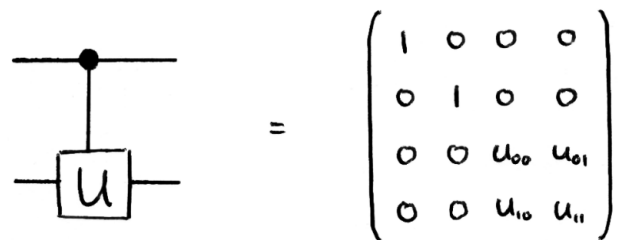
$$XX_{\phi} = \begin{pmatrix} 1 & 0 & 0 & -ie^{i\phi} \\ 0 & 1 & -i & 0 \\ 0 & -i & 1 & 0 \\ -ie^{i\phi} & 0 & 0 & 1 \end{pmatrix}$$



Fredkin Gate (CSWAP)



Controlled - U



References / Good Resources

A. Barenco, C.H. Bennett, R. Cleve, D.P. DiVincenzo, N. Margolus, P. Shor

T. Shenton, J. Smolin, H. Weinfurter - Elementary Gates for Quantum Computation (1995)

V.V. Shende, I.L. Markov - On the CNOT cost of the Toffoli Gate (2008)